GCE

FURTHER MATHEMATICS

UNIT 3: FURTHER MECHANICS A

SAMPLE ASSESSMENT MATERIALS

(1 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. By burning a charge, a cannon fires a cannon ball of mass 12 kg horizontally. As the cannon ball leaves the cannon, its speed is 600 ms^{-1}. The recoiling part of the cannon has a mass of 1600 kg.

   (a) Determine the speed of the recoiling part immediately after the cannon ball leaves the cannon. [3]

   (b) Find the energy created by the burning of the charge. State any assumption you have made in your solution and briefly explain how the assumption affects your answer. [5]

   (c) Calculate the constant force needed to bring the recoiling part to rest in 1.2 m. State, with a reason, whether your answer is an overestimate or an underestimate of the actual force required. [4]

2. A particle *P*, of mass 3 kg, is attached to a fixed point *O* by a light inextensible string of length 4 m. Initially, particle *P* is held at rest at a point which is $2\sqrt{3}$ m horizontally from *O*. It is then released and allowed to fall under gravity.

   (a) Show that the speed of *P* when it first begins to move in a circle is $\sqrt{3g}$. [4]

   (b) In the subsequent motion, when the string first makes an angle of 45° with the downwards vertical,

      (i) calculate the speed *v* of *P*,

      (ii) determine the tension in the string. [8]

3. At time $t = 0$ s, the position vector of an object *A* is $\mathbf{i}$ m and the position vector of another object *B* is $3\mathbf{i}$ m. The constant velocity vector of *A* is $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ ms^{-1} and the constant velocity vector of *B* is $\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ ms^{-1}. Determine the value of *t* when *A* and *B* are closest together and find the least distance between *A* and *B*. [9]

4. Relative to a fixed origin O, the position vector $\mathbf{r}$ m at time t s of a particle P, of mass 0.4 kg, is given by

$$\mathbf{r} = e^{2t}\mathbf{i} + \sin(2t)\mathbf{j} + \cos(2t)\mathbf{k}.$$

(a) Show that the velocity vector $\mathbf{v}$ and the position vector $\mathbf{r}$ are never perpendicular to each other. [6]

(b) Given that the speed of P at time t is v ms^{-1}, show that

$$v^2 = 4e^{4t} + 4.$$ [2]

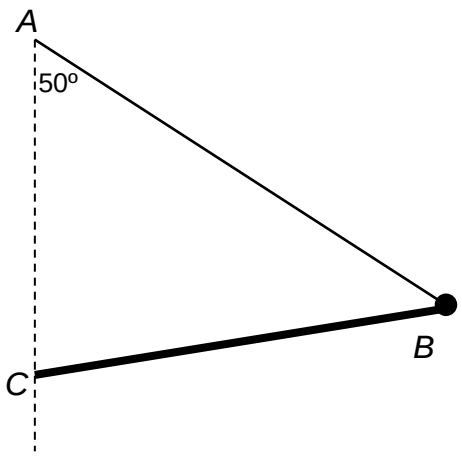
(c) Find the kinetic energy of P at time t. [1]

(d) Calculate the work done by the force acting on P in the interval $0 < t < 1$. [2]

(e) Determine an expression for the rate at which the force acting on P is working at time t. [2]

5. A particle of mass m kg is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set in motion such that it moves in a horizontal circle of radius 2 m with constant speed 4.8 ms^{-1}.
Calculate the angle the string makes with the vertical. [6]

6.



A particle of mass 5 kg is attached to a string *AB* and a rod *BC* at the point *B*. The string *AB* is light and elastic with modulus λ N and natural length 2 m. The rod *BC* is light and of length 2 m. The end *A* of the string is attached to a fixed point and the end C of the rod is attached to another fixed point such that *A* is vertically above *C* with *AC* = 2 m. When the particle rests in equilibrium, *AB* makes an angle of 50º with the downward vertical.

(a) Determine, in terms of λ, the tension in the string *AB*. [3]

(b) Calculate, in terms of λ, the energy stored in the string *AB*. [2]

(c) Find, in terms of λ, the thrust in the rod *BC*. [4]

7. A vehicle of mass 6000 kg is moving up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{6}{49}$. The vehicle's engine exerts a constant power of P W. The constant resistance to motion of the vehicle is R N. At the instant the vehicle is moving with velocity $\frac{16}{5}$ ms^{-1}, its acceleration is 2 ms^{-2}. The maximum velocity of the vehicle is $\frac{16}{3}$ ms^{-1}.

Determine the value of P and the value of R. [9]